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ON NON-HOMOGENEOUS TERNARY CUBIC DIOPHANTINE EQUATION

$$w^2 + 2z^2 - 2wx - 4zx = x^3 - 3x^2$$

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ABSTRACT:

The non-homogeneous ternary cubic diophantine equation $w^2 + 2z^2 - 2wx - 4zx = x^3 - 3x^2$ is analyzed for its patterns of non-zero distinct integral solutions. A few relations between the solutions and special number patterns are presented.

KEYWORDS: Ternary cubic Non- Homogeneous cubic, Integral solutions

NOTATIONS:

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$P_n^r = \frac{n(n+1)(n(r-2)-(r-5))}{6}$$

$$CP_k^3 = \frac{k^3 + k}{2} \quad , \quad CP_k^{12} = k(2k^2 - 1) \quad , \; CP_k^{18} = \frac{18k^3 - 12k}{6}$$

INTRODUCTION:

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-12] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $w^2 + 2z^2 - 2wx - 4zx = x^3 - 3x^2$ representing non-homogeneous cubic with three unknowns for determining its infinitely many non-zero integral points. A few relations between the solutions and special number patterns are presented.

METHOD OF ANALYSIS:

The given non-homogeneous ternary cubic diophantine equation is

$$w^2+2z^2-2wx-4zx=x^3-3x^2 \quad (1)$$

We illustrate below the process of obtaining different sets of integer solutions to (1):

Set 1:

On completing the squares,(1) is written as

$$P^2+2Q^2=x^3 \quad (2)$$

where

$$P=w-x, Q=z-x \quad (3)$$

After some algebra,it is observed that (2) is satisfied by

$$P=m(m^2+2n^2), Q=n(m^2+2n^2) \quad (4)$$

and

$$x=m^2+2n^2 \quad (5)$$

From (4) and (3) ,we have

$$w=(m+1)(m^2+2n^2), z=(n+1)(m^2+2n^2) \quad (6)$$

Thus,(5) and (6) represent the integer solutions to (1).

Observations:

(i). $(m+1)(w^2-x^2)$ is a square multiple of 6

(ii). $w^3-z^3+(n-m)^3x^3=3(m-n)xzw$

(iii). $w^2+z^2+(n-m)^2x^2=2(n-m)x(z-w)+2wz$

(iv). $(n^3+1)w^3-z^3-(1+m)^3z^3=-3n(m+1)zw^2$

(v). $w^2+n^2w^2+(1+m)^2z^2=-2nw^2+2(m+1)(n+1)wz$

(vi). When $n=1$,the ratio given by

$$\frac{m^2w+12mP^3_{m-1}}{P^3_m} \text{ is a square multiple of 6}$$

Set 2:

Assuming (5) in (2) and employing the method of factorization ,one obtains

$$P+i\sqrt{2}Q=(m+i\sqrt{2}n)^3 \tag{7}$$

On equating the real and imaginary parts ,we have

$$P=m^3-6mn^2, Q=3m^2n-2n^3 \tag{8}$$

Using (8) in (3), note that

$$z=m^2+2n^2+3m^2n-2n^3, w=m^2+2n^2+m^3-6mn^2 \tag{9}$$

Thus,(5) and (9) represent the integer solutions to (1).

Observations:

(i). When $m=1$, note that

$w+z-2x+6P_n^5+t_{14,n}+5n$ is a cubical integer

(ii). When $n=1$, it is seen that

$w-x-2CP_m^3$ is divisible by 7

(iii). The ratio given by

$\dfrac{2CP_m^3+x-w-m}{m}$ is a square multiple of 6

(iv). When $m=1$,observe that

$z-x+CP_n^{12}$ is even

(v). When $m=1$, it is obtained that

$9(z-x)+6CP_n^{18}$ is divisible by 15

Set 3:

Write (2) as

$$P^2+2Q^2=x^3*1 \tag{10}$$

Consider 1 as

$$1=\frac{(1+i2\sqrt{2})(1-i2\sqrt{2})}{9} \tag{11}$$

Using (5) and (11) in (10) and employing the method of factorization,one has

$$P+i\sqrt{2}Q=\frac{(1+i2\sqrt{2})(m+i\sqrt{2}n)^3}{3} \tag{12}$$

Following the procedure as in Set 2 and replacing m by 3M,n by 3N,the integer solutions to (1) are given by

$$x = 9(M^2 + 2N^2),\ z = 9(2M^3 - 12MN^2 + 3M^2N - 2N^3 + M^2 + 2N^2),$$
$$w = 9(M^3 - 6MN^2 - 12M^2N + 8N^3 + M^2 + 2N^2)$$

Note:

One may also take 1 on the R.H.S. of (10) in general as

$$1 = \frac{(2r^2 - s^2 + i\,2rs\sqrt{2})(2r^2 - s^2 - i\,2rs\sqrt{2})}{(2r^2 + s^2)^2}$$

The repetition of the above process leads to different sets of solutions to (1) when r and s take different values.

Observations:

When $N = 1$, note that

(a). $w - 18P_M^5 + 54\,t_{6,M} \equiv 0\,(\mathrm{mod}\,18)$

(b). $36P_M^5 - 108\,M - w + 180 \equiv 0\,(\mathrm{mod}\,6)$

(c). $324P_M^3 - 432M - 3z \equiv 0\,(\mathrm{mod}\,6)$

(d). $108\,P_M^3 - z + 288 \equiv 0\,(\mathrm{mod}\,18)$

CONCLUSION:

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of homogeneous or non-homogeneous 3

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